

## APPENDICE FILTRI ATTIVI - LABORATORIO

**FORMULE UTILIZZATE PER IL CALCOLO DEI VALORI TEORICI DELLA FUNZIONE DI TRASFERIMENTO IN dB , DELLA FASE E VALORI DEI COEFFICIENTI UTILIZZATI.**

### FILTRI DEL I° ORDINE

#### Filtro LP (passa-basso)

$$G(j\omega) = \frac{1 + \frac{R_2}{R_1}}{1 + j\frac{f}{f_o}} \left\{ \begin{array}{l} |G(j\omega)| = \frac{1 + \frac{R_2}{R_1}}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2}} = \frac{|A_o|}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2}} \\ \varphi = -\operatorname{arctg} \frac{f}{f_o} \end{array} \right.$$

$$A_o = 1 + \frac{R_2}{R_1} \quad ; \quad \varphi = -\operatorname{arctg} \frac{f}{f_o}$$

$$|G(j\omega)|_{\text{dB}} = 20\log\left(1 + \frac{R_2}{R_1}\right) - 20\log\sqrt{1 + \left(\frac{f}{f_o}\right)^2} = 20\log|A_o| - 10\log\left[1 + \left(\frac{f}{f_o}\right)^2\right]$$

#### Filtro HP (passa-alto)

$$G(j\omega) = \frac{1 + \frac{R_2}{R_1}}{1 - j\frac{f_o}{f}} \left\{ \begin{array}{l} |G(j\omega)| = \frac{1 + \frac{R_2}{R_1}}{\sqrt{1 + \left(\frac{f_o}{f}\right)^2}} = \frac{|A_o|}{\sqrt{1 + \left(\frac{f_o}{f}\right)^2}} \\ \varphi = \operatorname{arctg} \frac{f_o}{f} \end{array} \right.$$

$$A_o = 1 + \frac{R_2}{R_1} \quad ; \quad \varphi = \operatorname{arctg} \frac{f_o}{f}$$

$$|G(j\omega)|_{\text{dB}} = 20\log\left(1 + \frac{R_2}{R_1}\right) - 20\log\sqrt{1 + \left(\frac{f_o}{f}\right)^2} = 20\log|A_o| - 10\log\left[1 + \left(\frac{f_o}{f}\right)^2\right]$$

## FILTRI DEL II° ORDINE

### Filtro LP (passa-basso)

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_o} \cdot \frac{f}{f_o}}$$

$$|G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2}}$$

$$\varphi = -\operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \quad \text{se } A_o > 0$$

$$\varphi = \pi - \operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \quad \text{se } A_o < 0$$

$$|G(j\omega)|_{\text{dB}} = 20 \log |A_o| - 10 \log \left\{ \left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2 \right\}$$

- Filtro a retroazione multipla alla Butterworth:  $A_o = -\frac{R_3}{R_1} \quad ; \quad \frac{1}{Q_o} = \sqrt{2}$
- Filtro VCVS alla Butterworth:  $A_o = 1 + \frac{R_4}{R_3} \quad ; \quad \frac{1}{Q_o} = \sqrt{2}$

### Filtro HP (passa-alto)

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f_o}{f}\right)^2 - j \frac{1}{Q_o} \cdot \frac{f_o}{f}}$$

$$|G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2}}$$

$$\varphi = \operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f}{f_o} - \frac{f_o}{f}} \quad \text{se } A_o > 0$$

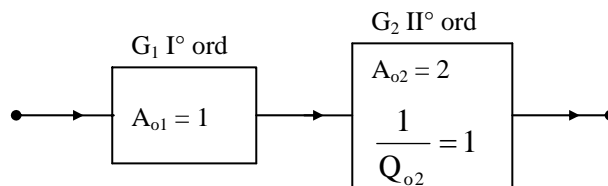
$$\varphi = \pi + \operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f}{f_o} - \frac{f_o}{f}} \quad \text{se } A_o < 0$$

$$|G(j\omega)|_{dB} = 20\log|A_o| - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2\right\}$$

- Filtro a retroazione multipla alla Butterworth:  $A_o = -\frac{C_1}{C_3}$  ;  $\frac{1}{Q_o} = \sqrt{2}$
- Filtro VCVS alla Butterworth:  $A_o = 1 + \frac{R_4}{R_3}$  ;  $\frac{1}{Q_o} = \sqrt{2}$

## FILTRI DI ORDINE SUPERIORE AL SECONDO VCVS A COMPONENTI UGUALI

### Filtro del III° ordine LP (passa-basso)



$$G_1(j\omega) = \frac{A_{o1}}{1 + j\frac{f}{f_o}}$$

$$\left\{ \begin{array}{l} |G_1(j\omega)| = \frac{|A_{o1}|}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2}} \\ \varphi_1 = -\arctg \frac{f}{f_o} \end{array} \right.$$

$$G_2(j\omega) = \frac{A_{o2}}{1 - \left(\frac{f}{f_o}\right)^2 + j\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}}$$

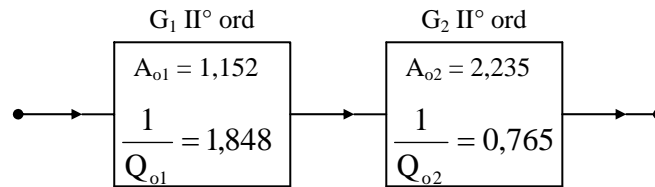
$$\left\{ \begin{array}{l} |G_2(j\omega)| = \frac{|A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}} \\ \varphi_2 = -\arctg \frac{\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}}{\frac{f}{f_o} - \left(\frac{f}{f_o}\right)^2} \end{array} \right.$$

$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| - 10\log\left[1 + \left(\frac{f}{f_o}\right)^2\right] - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\arctg \frac{f}{f_o} - \arctg \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtro del IV° ordine LP (passa-basso)



$$G_1(j\omega) = \frac{A_{o1}}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_{o1}} \cdot \frac{f}{f_o}}$$

$$|G_1(j\omega)| = \frac{|A_{o1}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2}}$$

$$\varphi_1 = -\arctg \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

$$G_2(j\omega) = \frac{A_{o2}}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_{o2}} \cdot \frac{f}{f_o}}$$

$$|G_2(j\omega)| = \frac{|A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}}$$

$$\varphi_2 = -\arctg \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

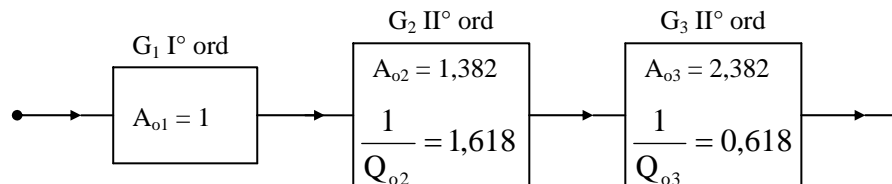
$$|G(j\omega)| = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} - \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtro del V° ordine LP (passa-basso)

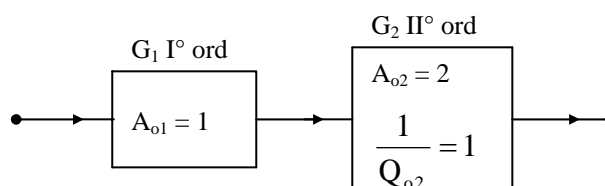


$$|G(j\omega)|_{\text{dB}} = 20\log|A_{o1}| + 20\log|A_{o2}| + 20\log|A_{o3}| - 10\log\left[1 + \left(\frac{f}{f_o}\right)^2\right] +$$

$$- 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o3}} \cdot \frac{f}{f_o}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 = -\operatorname{arctg} \frac{f}{f_o} - \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}} - \operatorname{arctg} \frac{\frac{1}{Q_{o3}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtro del III° ordine HP (passa-alto)



$$G_1(j\omega) = \frac{A_{o1}}{1 - j\frac{f_o}{f}} \left\{ \begin{array}{l} |G_1(j\omega)| = \frac{|A_{o1}|}{\sqrt{1 + \left(\frac{f_o}{f}\right)^2}} \\ \varphi_1 = \operatorname{arctg} \frac{f_o}{f} \end{array} \right.$$

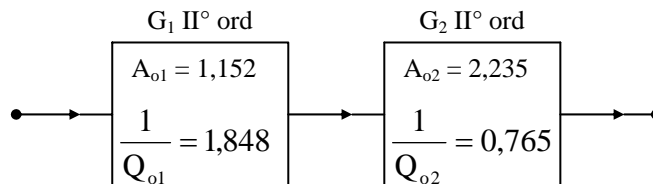
$$G_2(j\omega) = \frac{A_{o2}}{1 - \left(\frac{f_o}{f}\right)^2 - j \frac{1}{Q_{o2}} \cdot \frac{f_o}{f}} \quad \left\{ \begin{array}{l} |G_2(j\omega)| = \frac{|A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}} \\ \varphi_2 = \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}} \end{array} \right.$$

$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{1 + \left(\frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{\text{dB}} = 20 \log A_{o1} + 20 \log A_{o2} - 10 \log \left[ 1 + \left(\frac{f_o}{f}\right)^2 \right] - 10 \log \left\{ \left[ 1 - \left(\frac{f_o}{f}\right)^2 \right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2 \right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{f_o}{f} + \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

#### Filtro del IV° ordine HP (passa-alto)



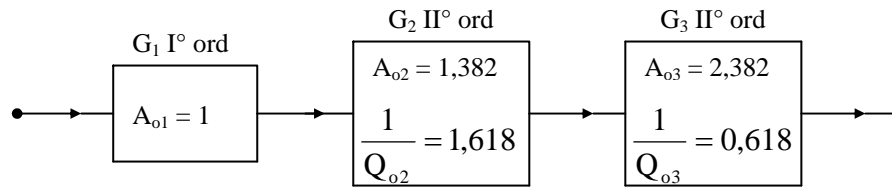
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{\text{dB}} = 20 \log A_{o1} + 20 \log A_{o2} +$$

$$- 10 \log \left\{ \left[ 1 - \left(\frac{f_o}{f}\right)^2 \right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2 \right\} - 10 \log \left\{ \left[ 1 - \left(\frac{f_o}{f}\right)^2 \right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2 \right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f}{f_o} - \frac{f_o}{f}} + \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

### Filtro del V° ordine HP (passa-alto)



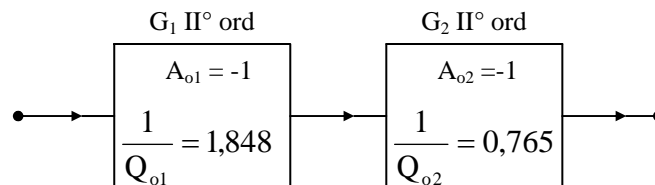
$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| + 20\log|A_{o3}| - 10\log\left[1 + \left(\frac{f_o}{f}\right)^2\right] +$$

$$- 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o3}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 = \arctg \frac{f}{f_o} + \arctg \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}} + \arctg \frac{\frac{1}{Q_{o3}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

### FILTRI A REAZIONE MULTIPLA DEL IV ORDINE ALLA BUTTERWORTH A GUADAGNO UNITARIO

#### Filtro LP (passa-basso)



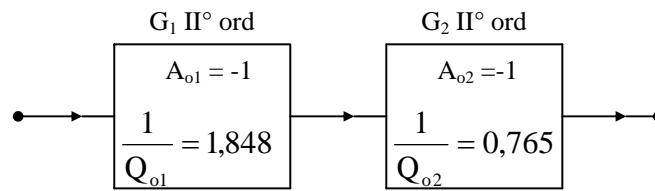
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$- 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \pi - \arctg \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} + \pi - \arctg \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}} = -\arctg \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} - \arctg \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

## Filtro LP (passa-basso)



$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \arctg \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} + \arctg \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

## FILTRI ALLA BESSEL

### Filtri del II° ordine VCVS a componenti uguali LP (passa-basso)

$$Q_o = 0,577 \quad ; \quad F_1 = 1,274 \quad ; \quad f_o = F_1 \cdot f_{-3dB} \quad ; \quad A_o = 1,267$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_o} \cdot \frac{f}{f_o}} \quad \left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2}} \\ \varphi = -\arctg \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20\log|A_o| - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2\right\}$$



**Filtri del II° ordine VCVS a componenti uguali HP (passa-alto)**

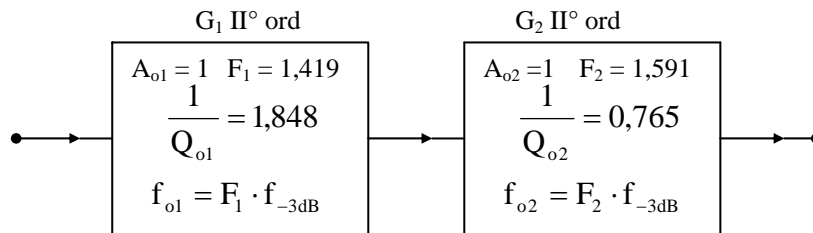
$$Q_o = 0,577 \quad ; \quad F_1 = 1,274 \quad ; \quad f_o = \frac{f_{-3dB}}{F_1} \quad ; \quad A_o = 1,267$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f_o}{f}\right)^2 - j \frac{1}{Q_o} \cdot \frac{f_o}{f}}$$

$$\left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2}} \\ \varphi = \text{arctg} \frac{\frac{1}{Q_o}}{\frac{f}{f_o} - \frac{f_o}{f}} \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20 \log |A_o| - 10 \log \left\{ \left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2 \right\}$$

**Filtri del IV° ordine VCVS a componenti uguali LP (passa-basso)**



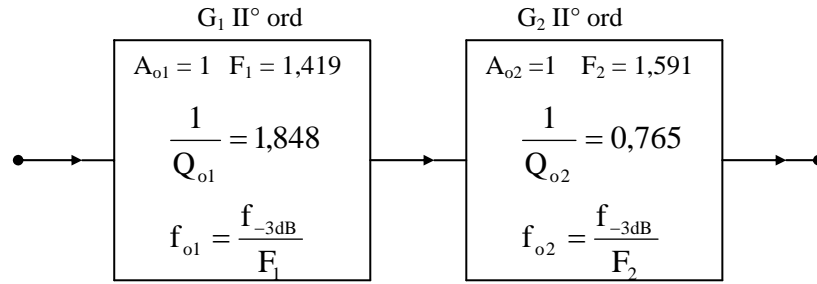
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20 \log |A_{o1}| + 20 \log |A_{o2}| +$$

$$-10 \log \left\{ \left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_o}\right)^2 \right\} - 10 \log \left\{ \left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_o}\right)^2 \right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\text{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f}{f_o} - \frac{f_o}{f}} - \text{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

### Filtri del IV° ordine VCVS a componenti uguali HP (passa-alto)



$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f}{f_o} - \frac{f_o}{f}} + \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

### Filtri del II° ordine a reazione multipla a guadagno unitario LP (passa-basso)

$$Q_o = 0,577 \quad ; \quad F_1 = 1,274 \quad ; \quad f_o = F_1 \cdot f_{-3dB} \quad ; \quad A_o = -1$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_o} \cdot \frac{f}{f_o}} \quad \left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2}} \\ \varphi = \pi - \operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \quad \text{perché } A_o < 0 \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20\log A_o - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2\right\}$$

**Filtri del II° ordine a reazione multipla a guadagno unitario HP (passa-alto)**

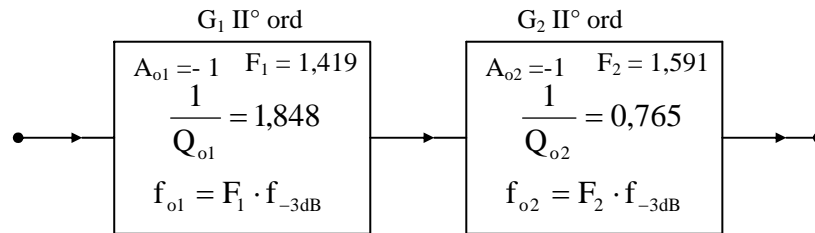
$$Q_o = 0,577 \quad ; \quad F_1 = 1,274 \quad ; \quad f_o = \frac{f_{-3dB}}{F_1} \quad ; \quad A_o = -1$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f_o}{f}\right)^2 - j \frac{1}{Q_o} \cdot \frac{f_o}{f}}$$

$$\left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2}} \\ \varphi = \pi + \operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \quad \text{perché } A_o < 0 \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20 \log |A_o| - 10 \log \left\{ \left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2 \right\}$$

**Filtri del IV° ordine a reazione multipla a guadagno unitario LP (passa-basso)**



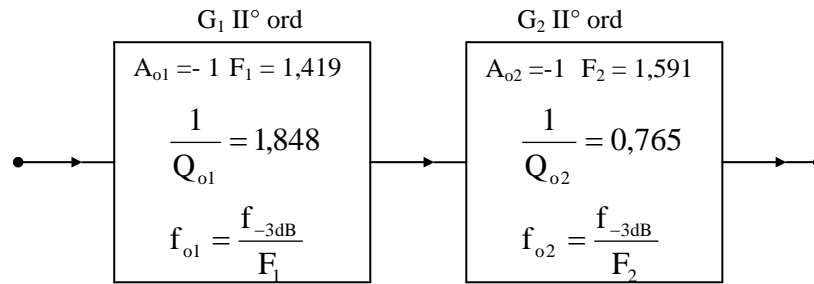
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_{o1}}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_{o2}}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20 \log |A_{o1}| + 20 \log |A_{o2}| +$$

$$-10 \log \left\{ \left[1 - \left(\frac{f}{f_{o1}}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}}\right)^2 \right\} - 10 \log \left\{ \left[1 - \left(\frac{f}{f_{o2}}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}}\right)^2 \right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} - \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

## Filtri del IV° ordine a reazione multipla a guadagno unitario HP (passa-alto)



$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f}{f_o} - \frac{f_o}{f}} + \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f}{f_o} - \frac{f_o}{f}}$$

## FILTRI ALLA CHEBYSHEV

### Filtri del II° ordine VCVS a componenti uguali LP (passa-basso)

$$Q_o = 1,129 \quad ; \quad F_1 = 0,907 \quad ; \quad f_o = F_1 \cdot f_c \quad ; \quad A_o = 2,114$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_o} \cdot \frac{f}{f_o}}$$

$$\left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2}} \\ \varphi = -\operatorname{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}} \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20\log|A_o| - 10\log\left\{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2\right\}$$

**Filtri del II° ordine VCVS a componenti uguali HP (passa-alto)**

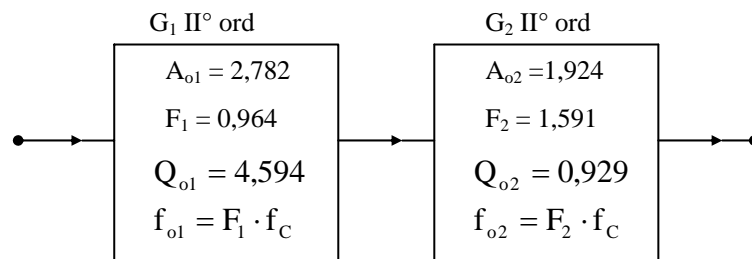
$$Q_o = 1,129 \quad ; \quad F_1 = 0,907 \quad ; \quad f_o = \frac{f_c}{F_1} \quad ; \quad A_o = 2,114$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f_o}{f}\right)^2 - j \frac{1}{Q_o} \cdot \frac{f_o}{f}}$$

$$\left\{ \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2}} \\ \varphi = \arctg \frac{1}{\frac{f}{f_o} - \frac{f_o}{f}} \end{array} \right.$$

$$|G(j\omega)|_{dB} = 20\log|A_o| - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f_o}{f}\right)^2\right\}$$

**Filtri del IV° ordine VCVS a componenti uguali LP (passa-basso)**



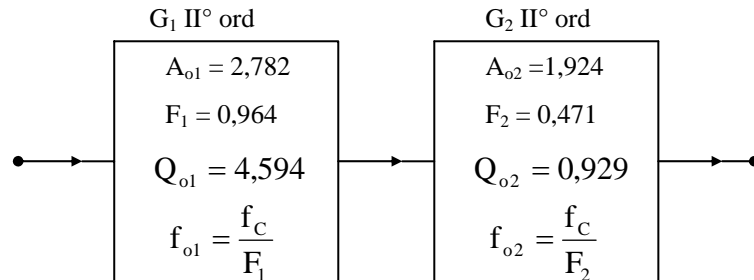
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f}{f_{o1}}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f}{f_{o2}}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f}{f_{o1}}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f}{f_{o2}}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\operatorname{arctg} \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}} - \operatorname{arctg} \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtri del IV° ordine VCVS a componenti uguali HP (passa-alto)



$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{01}| \cdot |A_{02}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{01}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{02}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{\text{dB}} = 20\log|A_{01}| + 20\log|A_{02}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{01}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{02}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}} + \operatorname{arctg} \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtri del II° ordine a reazione multipla a guadagno unitario LP (passa-basso)

$$Q_o = 1,129 \quad ; \quad F_1 = 0,907 \quad ; \quad f_o = F_1 \cdot f_c \quad ; \quad A_o = -1$$

$$G(j\omega) = \frac{A_o}{1 - \left(\frac{f}{f_o}\right)^2 + j \frac{1}{Q_o} \cdot \frac{f}{f_o}}$$

$$|G(j\omega)| = \frac{|A_o|}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left(\frac{1}{Q_o} \cdot \frac{f}{f_o}\right)^2}}$$

$$\varphi = \pi - \operatorname{arctg} \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}} \quad \text{perché } A_o < 0$$

$$|G(j\omega)|_{dB} = 20\log A_o - 10\log \left\{ \left[ 1 - \left( \frac{f}{f_o} \right)^2 \right]^2 + \left( \frac{1}{Q_o} \cdot \frac{f}{f_o} \right)^2 \right\}$$

**Filtri del II° ordine a reazione multipla a guadagno unitario HP (passa-alto)**

$$Q_o = 1,129 \quad ; \quad F_1 = 0,907 \quad ; \quad f_o = \frac{f_c}{F_1} \quad ; \quad A_o = -1$$

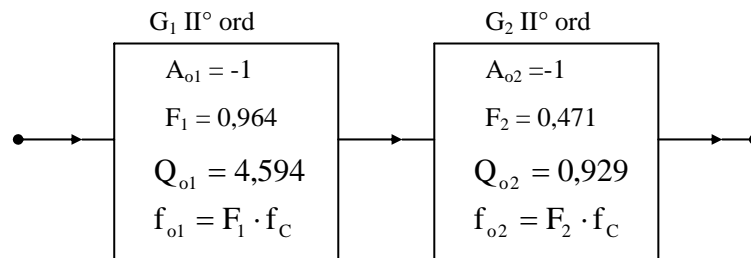
$$G(j\omega) = \frac{A_o}{1 - \left( \frac{f_o}{f} \right)^2 - j \frac{1}{Q_o} \cdot \frac{f_o}{f}}$$

$$|G(j\omega)| = \frac{|A_o|}{\sqrt{\left[ 1 - \left( \frac{f_o}{f} \right)^2 \right]^2 + \left( \frac{1}{Q_o} \cdot \frac{f_o}{f} \right)^2}}$$

$$\varphi = \pi + \operatorname{arctg} \frac{Q_o}{\frac{f}{f_o} - \frac{f_o}{f}} \quad \text{perché } A_o < 0$$

$$|G(j\omega)|_{dB} = 20\log|A_o| - 10\log \left\{ \left[ 1 - \left( \frac{f_o}{f} \right)^2 \right]^2 + \left( \frac{1}{Q_o} \cdot \frac{f_o}{f} \right)^2 \right\}$$

**Filtri del IV° ordine a reazione multipla a guadagno unitario LP (passa-basso)**



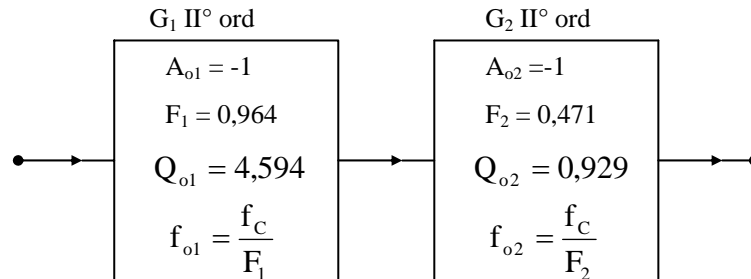
$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[ 1 - \left( \frac{f}{f_{o1}} \right)^2 \right]^2 + \left( \frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}} \right)^2} \cdot \sqrt{\left[ 1 - \left( \frac{f}{f_{o2}} \right)^2 \right]^2 + \left( \frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}} \right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log \left\{ \left[ 1 - \left( \frac{f}{f_{o1}} \right)^2 \right]^2 + \left( \frac{1}{Q_{o1}} \cdot \frac{f}{f_{o1}} \right)^2 \right\} - 10\log \left\{ \left[ 1 - \left( \frac{f}{f_{o2}} \right)^2 \right]^2 + \left( \frac{1}{Q_{o2}} \cdot \frac{f}{f_{o2}} \right)^2 \right\}$$

$$\varphi = \varphi_1 + \varphi_2 = -\operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} - \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

**Filtri del IV° ordine a reazione multipla a guadagno unitario HP (passa-alto)**



$$G(j\omega) = |G_1(j\omega)| \cdot |G_2(j\omega)| = \frac{|A_{o1}| \cdot |A_{o2}|}{\sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2} \cdot \sqrt{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2}}$$

$$|G(j\omega)|_{dB} = 20\log|A_{o1}| + 20\log|A_{o2}| +$$

$$-10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o1}} \cdot \frac{f_o}{f}\right)^2\right\} - 10\log\left\{\left[1 - \left(\frac{f_o}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{o2}} \cdot \frac{f_o}{f}\right)^2\right\}$$

$$\varphi = \varphi_1 + \varphi_2 = \operatorname{arctg} \frac{\frac{1}{Q_{o1}}}{\frac{f_o}{f} - \frac{f}{f_o}} + \operatorname{arctg} \frac{\frac{1}{Q_{o2}}}{\frac{f_o}{f} - \frac{f}{f_o}}$$



## FILTRI PASSA BANDA

$$G(j\omega) = \frac{\frac{A_o}{Q_o}}{\frac{1}{Q_o} - j\left(\frac{f_o}{f} - \frac{f}{f_o}\right)}$$

$$\left. \begin{array}{l} |G(j\omega)| = \frac{|A_o|}{Q_o} \\ \sqrt{\frac{1}{Q_o^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2} \\ \varphi = \arctg\left[Q_o \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right] \quad \text{se } A_o > 0 \\ \varphi = \pi + \arctg\left[Q_o \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right] \quad \text{se } A_o < 0 \end{array} \right\}$$

$$|G(j\omega)|_{dB} = 20\log\frac{|A_o|}{Q_o} - 20\log\sqrt{\frac{1}{(Q_o)^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2}$$

### Filtro del II° ordine VCVS a componenti uguali BP (passa-banda)

$$|G(j\omega)|_{dB} = 20\log\frac{|A_o|}{Q_o} - 20\log\sqrt{\frac{1}{(Q_o)^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2} \quad ; \quad \varphi = \arctg\left[Q_o \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right]$$

### Filtro del II° ordine a reazione multipla $Q_o < 10$ BP (passa-banda)

$$|G(j\omega)|_{dB} = 20\log\frac{|A_o|}{Q_o} - 20\log\sqrt{\frac{1}{(Q_o)^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2} \quad ; \quad \varphi = \pi + \arctg\left[Q_o \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right]$$

### Filtro del II° ordine a reazione multipla $Q_o > 10$ BP (passa-banda)

$$|G(j\omega)|_{dB} = 20\log\frac{|A_o|}{Q_o} - 20\log\sqrt{\frac{1}{(Q_o)^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2} \quad ; \quad \varphi = \arctg\left[Q_o \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right]$$

### Filtro BP a reazione multipla con roll-off asintotico di 40dB/decade

$$Q_{ot} = 5 \quad ; \quad |A_o| = 9 \quad ; \quad |A_{o1}| = |A_{o2}| = 3 \quad : \quad |A_o| = |A_{o1}| \cdot |A_{o2}| = 9$$

Il  $Q_o$  di ogni cella sarà:  $Q_o = Q_{o1} = Q_{o2} = Q_{ot} \sqrt{\sqrt{2}-1} = 5 \cdot \sqrt{\sqrt{2}-1} = 3,22$

$$|G(j\omega)|_{dB} = 20\log\frac{|A_{o1}|}{Q_{o1}} + 20\log\frac{|A_{o2}|}{Q_{o2}} - 20\log\sqrt{\frac{1}{(Q_{o1})^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2} - 20\log\sqrt{\frac{1}{(Q_{o2})^2} + \left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2}$$

$$\varphi = \varphi_1 + \varphi_2 = \arctg\left[Q_{o1} \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right] + \arctg\left[Q_{o2} \cdot \left(\frac{f_o}{f} - \frac{f}{f_o}\right)\right]$$

## FILTRI A REIEZIONE DI BANDA

### Filtro a reiezione di banda a reazione multipla

$$A_o = -\frac{R_6}{R_5} \quad \text{guadagno del filtro elimina banda}$$

$Q_o$  e  $f_o$  parametri del filtro passa banda

$$G(s) = \frac{A_o(-\omega^2 + \omega_o^2)}{-\omega^2 + j\frac{\omega_o}{Q_o}\omega + \omega_o^2} = \frac{A_o}{\frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2} + j\frac{\omega_o}{Q_o} \cdot \frac{\omega}{\omega_o^2 - \omega^2}} = \frac{A_o}{1 + j\frac{1}{Q_o} \cdot \frac{1}{\frac{\omega_o^2 - \omega^2}{\omega\omega_o}}} = \frac{A_o}{1 + j\frac{1}{Q_o} \cdot \frac{1}{\frac{f_o}{f} - \frac{f}{f_o}}}$$

$$|G(s)|_{dB} = \frac{|A_o|}{\sqrt{1 + \frac{1}{Q_o^2} \cdot \frac{1}{\left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2}}} \quad ; \quad \varphi = \pi - \arctg\frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

### Filtro a reiezione di banda con rete a doppio T e $Q_o$ variabile

$$f_o = 4\text{KHz} \quad ; \quad |A_o| = 1 \quad ; \quad Q_o = \frac{1}{4 \cdot (1-K)} \quad ; \quad k = 0 ; 0,25 ; 0,5 ; 0,75 ; 1$$

$$|G(s)|_{dB} = \frac{|A_o|}{\sqrt{1 + \frac{1}{Q_o^2} \cdot \frac{1}{\left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2}}} \quad ; \quad \varphi = \pi - \arctg\frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

**Filtro a reiezione di banda con rete differenziatore a ponte,  $Q_o$  variabile e taratura della frequenza di reiezione**

$$f_o = 4\text{KHz} \quad ; \quad |A_o| = 1 \quad ; \quad Q_o = \frac{\sqrt{3R_1R_2}}{3(2R_1 + R_2)(1-k)} \quad ; \quad k = 0 ; 0,25 ; 0,5 ; 0,75 ; 1$$

$$|G(s)|_{\text{dB}} = \frac{|A_o|}{\sqrt{1 + \frac{1}{Q_o^2} \cdot \frac{1}{\left(\frac{f_o}{f} - \frac{f}{f_o}\right)^2}}} \quad ; \quad \varphi = \pi - \text{arctg} \frac{\frac{1}{Q_o}}{\frac{f_o}{f} - \frac{f}{f_o}}$$

**FILTRI A BANDA LARGA**

**Filtro passa banda ed elimina banda a larga banda alla Butterworth**

$A_{oL}$  ,  $f_{oL}$  e  $Q_{oL}$  parametri del filtro passa basso

$A_{oH}$  ,  $f_{oH}$  e  $Q_{oH}$  parametri del filtro passa alto

$$|G(j\omega)|_{\text{dB}} = 20\log|A_{oL}| - 10\log\left\{\left[1 - \left(\frac{f}{f_{oL}}\right)^2\right]^2 + \left(\frac{1}{Q_{oL}} \cdot \frac{f}{f_{oL}}\right)^2\right\} +$$

$$+ 20\log|A_{oH}| - 10\log\left\{\left[1 - \left(\frac{f_{oH}}{f}\right)^2\right]^2 + \left(\frac{1}{Q_{oH}} \cdot \frac{f_{oH}}{f}\right)^2\right\}$$

$$\varphi = -\text{arctg} \frac{\frac{1}{Q_{oL}}}{\frac{f_{oL}}{f} - \frac{f}{f_{oL}}} - \text{arctg} \frac{\frac{1}{Q_{oH}}}{\frac{f}{f_{oH}} - \frac{f_{oH}}{f}}$$

**FILTRO PASSA TUTTO (CIRCUITO SFASATORE)**

**Sfasamento positivo (in anticipo)**       $\varphi = \pi - 2\text{arctg}(2\pi fRC) = \pi - 2\text{arctg}(360^\circ fRC)$

**Sfasamento negativo (in ritardo)**       $\varphi = -2\text{arctg}(2\pi fRC) = -2\text{arctg}(360^\circ fRC)$